

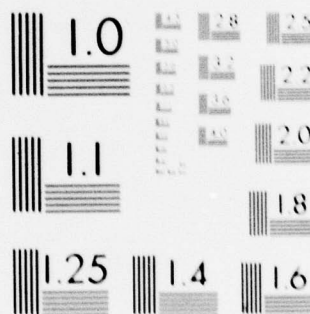
AD-A077 833 CENTER FOR NAVAL ANALYSES ALEXANDRIA VA NAVAL WARFARE--ETC F/G 15/3  
A CLASS OF COMMUTATIVE MARKOV MATRICES.(U)

UNCLASSIFIED CNA-PP-259 NOV 79 D V GLASS , I HSU , W R NUNN

NL

| OF |  
AD  
A077833

END  
DATE  
FILMED  
1-80  
DDC



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

AD A 077833

14 CNA-PP-11  
PROFESSIONAL PAPER 259 (November 1979)

LEVEL #

12 23

6  
**A CLASS OF COMMUTATIVE  
MARKOV MATRICES**

10  
David V. Glass  
Ih-Ching Hsu  
Walter R. Nunn  
David A. Perin

DDC FILE COPY

DDC  
RECEIVED  
DEC 7 1979  
A



**CENTER FOR NAVAL ANALYSES**

2000 North Beauregard Street, Alexandria, Virginia 22311

DISTRIBUTION STATEMENT A  
Approved for public release;  
Distribution Unlimited

70 127  
252950

046

4B

The ideas expressed in this paper are those of the authors.  
The paper does not necessarily represent the views of either the  
Center for Naval Analyses or the Department of Defense.

PROFESSIONAL PAPER 259 / November 1979

## A CLASS OF COMMUTATIVE MARKOV MATRICES

David V. Glass  
Ih-Ching Hsu  
Walter R. Nunn  
David A. Perin



*Naval Warfare Analysis Group*

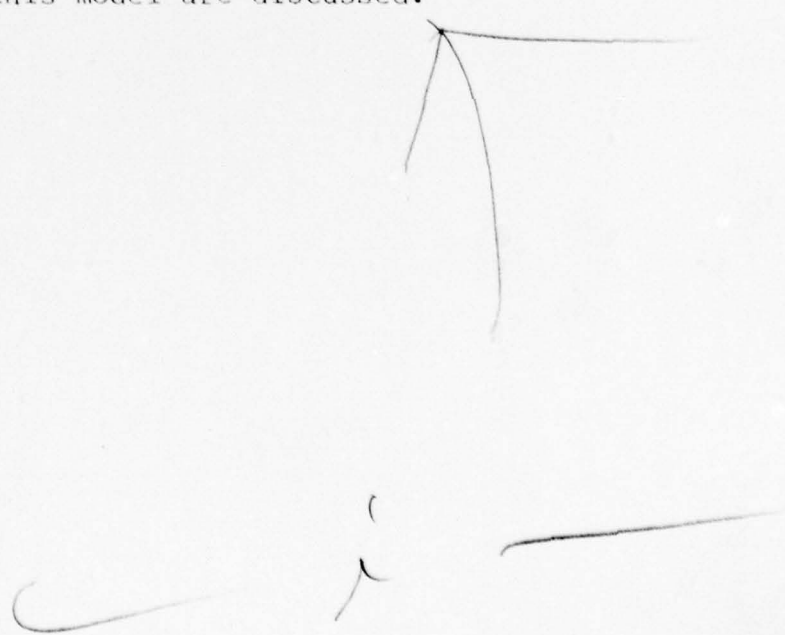
**CENTER FOR NAVAL ANALYSES**

2000 North Beauregard Street, Alexandria, Virginia 22311

Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DDC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/	
Availability Codes	
Dist.	Avail and/or special
A	

# ABSTRACT

~~We examine~~ a class of Markov matrices which arise in a simple model of a defense system. *HAS BEEN EXAMINED*  
The model illustrates a Markov chain which is not time-homogeneous but is still amenable to analytic treatment. The matrices are shown to be commutative and the class is shown to be closed under matrix multiplication. The matrices are also shown to be diagonalizable and the eigenvectors have a simple form, namely composed of elements of Pascal's triangle. A description of the defense system model is given and the implications of the mathematical results for this model are discussed.





## INTRODUCTION

A defense system model, described later, leads to the formulation of a class of matrices with parameters  $n$  and  $p$ , and with elements  $A_{ij}$ , where

$$A_{ij} = \begin{cases} 0 & \text{if } i < j \\ \binom{i}{j} p^{i-j} q^j & \text{if } i \geq j \end{cases}$$

for given  $p$  and  $q$  with  $0 \leq p \leq 1$  and  $1 = p+q$ . When  $i \geq j$ ,  $A_{ij}$  represents the probability of  $j$  failures in  $i$  Bernoulli trials, given  $q$ , the probability of a failure in a single trial. For simplicity, this class of matrices will be referred to as type  $\alpha$ .

For type  $\alpha$  matrices, we establish a proof of multiplicative commutativity by the diagonalization process via eigenvalues and eigenvectors. In doing so, we discover the intriguing result that the complete set of linearly independent eigenvectors forms a lower triangular matrix whose rows are those of Pascal's triangle. The eigenvectors are independent of  $p$  and  $q$ . Also, we show that the class of matrices of type  $\alpha$  is closed under matrix multiplication.

After the derivation of these mathematical results, the rest of the paper is devoted to the explanation of the role of type  $\alpha$  matrices in the model, and to the interpretation of the mathematical results for this model.

#### MATHEMATICAL RESULTS

##### Theorem 1.

For any positive integer  $n$ , let  $A$  be an  $(n+1)$  by  $(n+1)$  matrix of type  $\alpha$ , with  $p \geq 0$ ,  $q \geq 0$  and  $p+q = 1$ .

Then  $A$  is diagonalizable with  $A = SDS^{-1}$ , where

$D = \text{Diag } \{1, q, \dots, q^n\}$  and, independent of  $p$  and  $q$ ,  $S$  is simply a lower triangular matrix whose rows coincide with those of Pascal's triangle (if presented in the form of a right triangle).

##### Proof.

Since  $1, q, q^2, \dots, q^n$ , with,  $0 \leq q \leq 1$ , are the  $n+1$  distinct eigenvalues of matrix  $A$ ,  $A$  can be diagonalized to  $D = \text{Diag } \{1, q, \dots, q^n\}$

Mathematical induction on  $n$  will be used to prove  $AS=SD$ . From here on, appropriate subscripts will be used to indicate sizes of matrices under investigation; therefore:



$$A_{n+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & - & - & - & 0 \\ p & q & 0 & 0 & - & - & - & 0 \\ p^2 & 2pq & q^2 & 0 & - & - & - & 0 \\ | & & & & & & & | \\ | & & & & & & & | \\ | & & & & & & & | \\ \binom{n}{0} p^n & \binom{n}{1} p^{n-1} q & \binom{n}{2} p^{n-2} q^2 & - & - & - & - & q^n \end{bmatrix}$$

and

$$S_{n+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & - & - & - & 0 \\ 1 & 1 & 0 & 0 & - & - & - & 0 \\ 1 & 2 & 1 & 0 & - & - & - & 0 \\ | & & & & & & & | \\ | & & & & & & & | \\ | & & & & & & & | \\ \binom{n}{0} & \binom{n}{1} & \binom{n}{2} & - & - & - & - & \binom{n}{n} \end{bmatrix}$$

When  $n=1$ ,  $A_2 = \begin{bmatrix} 1 & 0 \\ p & q \end{bmatrix}$  and  $S_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ .

It can be easily verified that  $A_2 S_2 = S_2 D_2$ .

Assuming  $A_n S_n = S_n D_n$ , we want to establish  $A_{n+1} S_{n+1} = S_{n+1} D_{n+1}$ . Matrix block multiplication yields:

$$A_{n+1} S_{n+1} = \left[ \begin{array}{c|c} A_n S_n & \begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \end{array} \\ \hline [p^n, \binom{n}{1} p^{n-1} q, \dots, S_n] + q^n [\binom{n}{0}, \binom{n}{1}, \dots, \binom{n}{n-1}] & q^n \end{array} \right]$$

and

$$S_{n+1} D_{n+1} = \left[ \begin{array}{c|c} S_n D_n & \begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \end{array} \\ \hline \binom{n}{0}, \binom{n}{1} q, \dots, \binom{n}{n-1} q^{n-1} & q^n \end{array} \right]$$

Since  $A_n S_n = S_n D_n$ , all that remains to show, after comparison of  $A_{n+1} S_{n+1}$  with  $S_{n+1} D_{n+1}$ , is that:

$$[(\binom{n}{0}p^n, (\binom{n}{1}p^{n-1}q, \dots, (\binom{n}{n-1}pq^{n-1})] [S_n] + q^n[(\binom{n}{0}), (\binom{n}{1}), \dots,$$

$$(\binom{n}{n-1})] = [(\binom{n}{0}), (\binom{n}{1}q, \dots, (\binom{n}{n-1}q^{n-1})] \quad (1)$$

To establish the equality, we first expand a typical, say  $(r+1)$ st, term of the R.H.S. of equation 1:

$$q^r \binom{n}{r}, \text{ where } r = 0, 1, 2, \dots, n-1,$$

$$q^r \binom{n}{r} = q^r \binom{n}{r} (p+q)^{n-r} \text{ since } p+q = 1. \text{ Expanding } (p+q)^{n-r}:$$

$$= q^r \binom{n}{r} [(\binom{n-r}{0}p^{n-r}q^0 + (\binom{n-r}{1}p^{n-r-1}q^1 + \dots (\binom{n-r}{n-r-1}p^1q^{n-r-1} + (\binom{n-r}{n-r}p^0q^{n-r})]$$

$$= q^r [(\binom{n}{r})(\binom{n-r}{0}p^{n-r}q^0 + (\binom{n}{r})(\binom{n-r}{1}p^{n-r-1}q^1 + \dots (\binom{n}{r})(\binom{n-r}{n-r}p^0q^{n-r})]$$

$$= q^r [(\binom{n}{r})(\binom{r}{r}p^{n-r}q^0 + (\binom{n}{r+1})(\binom{r+1}{r}p^{n-r-1}q^1 + \dots (\binom{n}{n})(\binom{n}{r}p^0q^{n-r})],$$

since  $\binom{n}{r} \binom{n-r}{k} = \binom{n}{r+k} \binom{r+k}{r}$ , for  $k < n-r$

Thus,  $q^r \binom{n}{r}$  has the following expansion:

$$q^r \binom{n}{r} = \binom{n}{r} \binom{r}{r} p^{n-r} q^r + \binom{n}{r+1} \binom{r+1}{r} p^{n-r-1} q^{r+1} + \dots + \binom{n}{n} \binom{n}{r} p^0 q^n .$$

The formation of the L.H.S. of equation 1 indicates that its  $(r+1)$ st term is the sum of  $q^n \binom{n}{r}$  and a dot product. While  $q^n \binom{n}{r}$  is simply the  $(r+1)$ st element in  $q^n [ \binom{n}{0}, \binom{n}{1}, \dots, \binom{n}{n-1} ]$ , the dot product is that of the first  $n$  elements of the  $(n+1)$ st row in  $A_{n+1}$  and the  $(r+1)$ st column in  $S_n$ , this column being equal to  $[0, \dots, 0, \binom{r}{r}, \binom{r+1}{r}, \dots, \binom{n-1}{r}]$ . After expansion, the dot product is equal to  $\binom{n}{r} \binom{r}{r} p^{n-r} q^r + \binom{n}{r+1} \binom{r+1}{r} p^{n-r-1} q^{r+1} + \dots + \binom{n}{n-1} \binom{n-1}{r} p q^{n-1}$ . This sum plus  $q^n \binom{n}{r}$  is indeed identical to the earlier expansion for the  $(r+1)$ st term of the R.H.S. of equation 1. The equality  $A_{n+1} S_{n+1} = S_{n+1} D_{n+1}$  is now established, and this completes the proof.



Theorem 2. Multiplicative commutativity holds for square matrices of type  $\alpha$ .

Proof. Let A and B be two  $(n+1) \times (n+1)$  matrices of type  $\alpha$ . Then, by Theorem 1, A and B can be diagonalized with  $A = SDS^{-1}$  and  $B = SD'S^{-1}$  where D and D' are diagonal matrices and S is the lower triangular matrix described in Theorem 1. The commutativity of A and B now is evident, since:

$$AB = SDS^{-1}SD'S^{-1} = SDD'S^{-1} = SD'DS^{-1} = BA.$$

We comment that the matrices S, with  $i, j$ th element equal to  $\binom{i}{j}$ , are seen in combinatoric settings, e.g., reference 1, page 45. Reference 1 also proves that the  $i, j$ th element of  $S^{-1}$  is given by  $(-1)^{i+j} \binom{i}{j}$ , so that  $S^{-1}$  is obtained directly from S by inserting "-" at appropriate locations. Hence, the spectral decomposition of  $A_{n+1}$  can be written down almost by inspection, i.e.,

$$A_{n+1} = \underline{u}_1 \underline{v}_1 + q \underline{u}_2 \underline{v}_2 + \dots + q^n \underline{u}_{n+1} \underline{v}_{n+1}$$



where the column vector  $\underline{u}_k$  is the  $k$ th column of  $S_{n+1}$

and the row vector  $\underline{v}_k$  is the  $k$ th row of  $S_{n+1}^{-1}$ .

A direct verification of Theorem 2 is instructive.

Suppose that the  $i, j$ th element of  $A$  is  $\binom{i}{j} q_1^j p_1^{i-j}$

and the  $j, k$ th element of  $B$  is  $\binom{j}{k} q_2^k p_2^{j-k}$ . Then the

$i, k$ th element of  $AB$  is given by

$$\begin{aligned} & \sum_{j=0}^n \binom{i}{j} \binom{j}{k} q_1^j q_2^k p_1^{i-j} p_2^{j-k} \\ &= \binom{i}{k} (q_1 q_2)^k \sum_j \binom{i-k}{j-k} (q_1 p_2)^{j-k} p_1^{i-j} \\ &= \binom{i}{k} (q_1 q_2)^k (1 - q_1 q_2)^{i-k} \end{aligned}$$

where we have used the identity  $\binom{i}{j} \binom{j}{k} = \binom{i}{k} \binom{i-k}{j-k}$

and recognized the binomial expansion of  $(p_1 + q_1 p_2)^{i-k}$ .

Clearly, the steps remain valid with  $q_1$  and  $q_2$  inter-

changed, and the commutativity of  $A$  and  $B$  follows.

Further examination of the proof shows that we have also shown that the class of matrices of type  $\alpha$  is closed under matrix multiplication. More precisely, Theorem 3. If A and B are matrices of type  $\alpha$  of the same order and with parameters  $q_1, q_2$ , respectively, then AB is of type  $\alpha$  with parameter  $q_1 q_2$ .

Theorem 3 will be useful for providing computational simplification in our model.

#### MODEL DESCRIPTION

The foregoing class of matrices arise in the context of a defense system model in which a cluster of  $n$  missiles attempts to penetrate several lines of defense. The number of missiles surviving the first line of defense is Binomially distributed with parameters  $n, q_1$ , i.e., there are  $i$  survivors with probability  $\binom{n}{i} q_1^i p_1^{n-i}$ ,  $i=0,1,\dots,n$ , and  $q_1 = 1-p_1$ . Suppose that  $n_1$  missiles survive the first line of defense. We assume that the number of missiles surviving the second line of defense is Binomially

distributed with parameters  $n_1, q_2$ , and etc.<sup>a</sup>

In short, we assume that the lines of defense are independent and produce attrition in accordance with a Binomial distribution - each line of defense having its own "success probability"  $p_k$  and associated

$$q_k = 1 - p_k.$$

It is convenient to regard the missiles' passage through a particular line of defense as a transition in a Markov chain (reference 2). For example, given  $n = 2$  and  $p_1 = 0.6$ , the matrix below is the transition matrix for penetration of the first line of defense.

		<u>Out</u>		
		0	1	2
<u>In</u>	0	1	0	0
	1	.6	.4	0
	2	.36	.48	.16

<sup>a</sup>It is a standard result in probability theory that if the the random variable  $X$ , given  $N$ , is distributed as Binomial with parameters  $N, p$ , and if  $N$  is distributed as Binomial with parameters  $N^*, p^*$ , then the unconditional distribution of  $X$  is Binomial with parameters  $N^*, pp^*$ . The proof is virtually identical with that of theorem 3. Hence we have a probabilistic proof that the number of survivors of the second line of defense is distributed as Binomial with parameters  $n, q_1 q_2$ , and an induction argument gives the general result for several lines of defense.

Thus, if 2 missiles enter this line of defense, the probability that 1 missile emerges is 0.48. We see that this matrix is just  $A_3$  defined in the preceeding section (with  $p = 0.6$ ). In general the number of incoming missiles might be a discrete random variable having probability distribution  $T$ , whereupon the number of missiles surviving the line of defense is found by pre-multiplying the transition matrix by the row vector  $T$ . In our 3x3 example, if the probabilities of having 0,1,2 incoming missiles were 0,.5,.5, respectively, then the computation would be

$$(0 \quad .5 \quad .5) \begin{bmatrix} 1 & 0 & 0 \\ .6 & .4 & 0 \\ .36 & .48 & .16 \end{bmatrix} = (.48 \quad .44 \quad .08)$$

so that the probability of having 0,1,2 missiles penetrate the first line of defense would be .48, .44, .08, respectively. This resultant vector serves as the input to the second line of defense, and by successive such vector-matrix multiplications the probability of having 0,1, or 2 missiles penetrate all the defenses can be calculated. In particular, it follows that if



there are  $L$  levels of defense and the transition matrix for each level is the matrix  $A_3(k)$ ,  $k=1,2,\dots,L$ , then the distribution of the number of penetrators is given by

$$(0 \quad .5 \quad .5) \prod_{k=1}^L A_3(k)$$

More generally, if there may be as many as  $n-1$  incoming missiles then the vector  $T$  will need to have  $n$  elements and the transition matrices will have to be of size  $n \times n$ . The reader should verify that the transition matrices for this general case are precisely the matrices  $A_n$  defined in the previous section, with  $p = p_k$  as appropriate. Thus, we have a Markov chain in discrete time which is not time-homogeneous, i.e., the transition matrices are not identical, and for which - as we shall see - closed form results are available.

#### DISCUSSION AND CONCLUSIONS

A non-trivial question is "Does the ordering of the lines of defense make any difference?". Mathematically, this is the same as asking if the ordering of the  $A_n(k)$  makes any difference in evaluating the



product  $\prod A_n(k)$ . It is well known that matrix multiplication is non-commutative in general, and since the  $A_n(k)$  matrices will not usually be identical (unless the  $p_k$ 's are identical) it is not obvious mathematically that these matrices will commute. However, the results in the previous section show that the  $A_n(k)$  matrices do commute and the ordering of the lines of defense makes no difference in this model.

The naive approach to evaluating the product  $\prod A_n(k)$  is to do it by brute force, performing the indicated matrix multiplications one at a time. A more sophisticated approach is to use theorem 1 and recognize that the product can be written as  $SDS^{-1}$  where  $D$  is the diagonal matrix whose  $i$ th diagonal entry is  $\prod q_k^i$ , requiring only two matrix multiplications. A still more sophisticated approach involves theorem 3, seeing that the product can be written down immediately using no matrix multiplications at all. This illustrates the practical utility of increased mathematical insight in analysis of our model.

Finally, we observe that our results for this model find application in a more general setting. For example, instead of missiles let the survivors be ships in a task force, and instead of levels of defense let the attrition occur because of a sequence of attacks on the task force. Intuitively, one feels that an optimal defense should be insensitive to the sequencing of attacks, even in elaborate models, so that there is some possibility of the kind of computational simplifications that we have seen in our simple model.

#### REFERENCES

1. Riordan, John, Combinatorial identities, John Wiley & Sons, Inc. 1968
2. Howard, Ronald A., Dynamic Probabilistic Systems, Vol. 1: Markov Models, Wiley, 1971

# CNA Professional Papers – 1973 to Present\*

- PP 103  
Friedheim, Robert L., "Political Aspects of Ocean Ecology" 48 pp., Feb 1973, published in *Who Protects the Oceans*, John Lawrence Margrove (ed.) (St. Paul: West Publ'g Co., 1974), published by the American Society of International Law AD 757 936
- PP 104  
Schick, Jack M., "A Review of James Cable, Gunboat Diplomacy Political Applications of Limited Naval Forces," 5 pp., Feb 1973, (Reviewed in the *American Political Science Review*, Vol. LXVI, Dec 1972)
- PP 105  
Corn, Robert J. and Phillips, Gary R., "On Optimal Correction of Gunfire Errors," 22 pp., Mar 1973, AD 761 674
- PP 106  
Stoloff, Peter H., "User's Guide for Generalized Factor Analysis Program (FACTAN)," 35 pp., Feb 1973, (Includes an addendum published Aug 1974) AD 758 824
- PP 107  
Stoloff, Peter H., "Relating Factor Analytically Derived Measures to Exogenous Variables," 17 pp., Mar 1973, AD 758 820
- PP 108  
McConnell, James M. and Kelly, Anne M., "Superpower Naval Diplomacy in the Indo-Pakistani Crisis," 14 pp., 5 Feb 1973, (Published, with revisions, in *Survival*, Nov/Dec 1973) AD 761 675
- PP 109  
Berghofer, Fred G., "Salaries—A Framework for the Study of Trend," 8 pp., Dec 1973, (Published in *Review of Income and Wealth*, Series 18, No. 4, Dec 1972)
- PP 110  
Augusta, Joseph, "A Critique of Cost Analysis," 9 pp., Jul 1973, AD 766 376
- PP 111  
Herrick, Robert W., "The USSR's 'Blue Belt of Defense' Concept: A Unified Military Plan for Defense Against Seaborne Nuclear Attack by Strike Carriers and Polaris/Poseidon SSBNs," 18 pp., May 1973, AD 766 375
- PP 112  
Ginsberg, Lawrence H., "ELF Atmosphere Noise Level Statistics for Project SANGUINE," 29 pp., Apr 1974, AD 786 969
- PP 113  
Ginsberg, Lawrence H., "Propagation Anomalies During Project SANGUINE Experiments," 5 pp., Apr 1974, AD 786 968
- PP 114  
Meloney, Arthur P., "Job Satisfaction and Job Turnover," 41 pp., Jul 1973, AD 766 410
- PP 115  
Silverman, Lester P., "The Determinants of Emergency and Elective Admissions to Hospitals," 146 pp., 18 Jul 1973, AD 766 377
- PP 116  
Rehm, Allan S., "An Assessment of Military Operations Research in the USSR," 19 pp., Sep 1973, (Reprinted from *Proceedings, 30th Military Operations Research Symposium (U)*, Secret Dec 1972) AD 770 116
- PP 117  
McWhite, Peter B. and Ratliff, H. Donald, "Defending a Logistics System Under Mining Attack," 24 pp., Aug 1976 (to be submitted for publication in *Naval Research Logistics Quarterly*), presented at 44th National Meeting, Operations Research Society of America, November 1973, AD A030 454  
\*University of Florida  
\*\*Research supported in part under Office of Naval Research Contract N00014-68-0273-0017
- PP 118  
Barfoot, C. Bernard, "Markov Duels," 18 pp., Apr 1973, (Reprinted from *Operations Research*, Vol. 22, No. 2, Mar-Apr 1974)
- PP 119  
Stoloff, Peter and Lockman, Robert F., "Development of Navy Human Relations Questionnaire," 2 pp., May 1974, (Published in *American Psychological Association Proceedings, 81st Annual Convention, 1973*) AD 779 240
- PP 120  
Smith, Michael W. and Schrimper, Ronald A., "Economic Analysis of the Intracity Dispersion of Criminal Activity," 30 pp., Jun 1974, (Presented at the Econometric Society Meetings, 30 Dec 1973) AD 780 538  
\*Economics, North Carolina State University
- PP 121  
Devine, Eugene J., "Procurement and Retention of Navy Physicians," 21 pp., Jun 1974, (Presented at the 49th Annual Conference, Western Economic Association, Las Vegas, Nev., 10 Jun 1974) AD 780 539
- PP 122  
Kelly, Anne M., "The Soviet Naval Presence During the Iraq-Kuwait Border Dispute: March-April 1973," 34 pp., Jun 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 780 592
- PP 123  
Petersen, Charles C., "The Soviet Port-Clearing Operation in Bangladesh, March 1972-December 1973," 35 pp., Jun 1974, (Published in Michael McGwire, et al. (eds) *Soviet Naval Policy: Objectives and Constraints*, (New York: Praeger Publishers, 1974) AD 780 540
- PP 124  
Friedheim, Robert L. and Jehn, Mary E., "Anticipating Soviet Behavior at the Third U.N. Law of the Sea Conference: USSR Positions and Dilemmas," 37 pp., 10 Apr 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 780 541
- PP 125  
Weinland, Robert G., "Soviet Naval Operations—Ten Years of Change," 17 pp., Aug 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 783 962
- PP 126 – Classified.
- PP 127  
Dagnich, George S., "The Soviet Union's Quest for Access to Naval Facilities in Egypt Prior to the June War of 1967," 64 pp., Jul 1974, AD 786 318
- PP 128  
Stoloff, Peter and Lockman, Robert F., "Evaluation of Naval Officer Performance," 11 pp., (Presented at the 82nd Annual Convention of the American Psychological Association, 1974) Aug 1974, AD 784 012
- PP 129  
Holen, Arlene and Morowitz, Stanley, "Partial Unemployment Insurance Benefits and the Extent of Partial Unemployment," 4 pp., Aug 1974, (Published in the *Journal of Human Resources*, Vol. IX, No. 3, Summer 1974) AD 784 010
- PP 130  
Dismukes, Bradford, "Roles and Missions of Soviet Naval General Purpose Forces in Wartime: PROSSBN Operation," 20 pp., Aug 1974, AD 786 320
- PP 131  
Weinland, Robert G., "Analysis of Gorchkov's *Navies in War and Peace*," 45 pp., Aug 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 786 318
- PP 132  
Kleinman, Samuel D., "Racial Differences in Hours Worked in the Market: A Preliminary Report," 77 pp., Feb 1975, (Paper read on 26 Oct 1974 at Eastern Economic Association Convention in Albany, N.Y.) AD A 005 517
- PP 133  
Squires, Michael L., "A Stochastic Model of Regime Change in Latin America," 42 pp., Feb 1975, AD A 007 912
- PP 134  
Root, R. M. and Cuniff, P. F., "A Study of the Shock Spectrum of a Two-Degree-of-Freedom Non-linear Vibratory System," 39 pp., Dec 1975, (Published in the condensed version of *The Journal of the Acoustic Society*, Vol 60, No. 6, Dec 1976, pp. 1314  
\*Department of Mechanical Engineering, University of Maryland
- PP 135  
Goudreau, Kenneth A.; Kuzmack, Richard A.; Wiedemann, Karen, "Analysis of Closure Alternatives for Naval Stations and Naval Air Stations," 47 pp., 3 Jun 1975 (Reprinted from "Hearing before the Subcommittee on Military Construction of the Committee on Armed Service," U.S. Senate, 93rd Congress, 1st Session, Part 2, 22 Jun 1973)
- PP 136  
Stallings, William, "Cybernetics and Behavior Therapy," 13 pp., Jun 1975
- PP 137  
Petersen, Charles C., "The Soviet Union and the Reopening of the Suez Canal: Miniclearing Operations in the Gulf of Suez," 30 pp., Aug 1975, AD A 015 376

\*CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the author at the Center for Naval Analyses, 2000 North Beauregard Street, Alexandria, Virginia 22311.



- PP 138  
Stallings, William, "BRIDGE: An Interactive Dialogue-Generation Facility," 5 pp., Aug 1975 (Reprinted from IEEE Transactions on Systems, Man, and Cybernetics, Vol. 5, No. 3, May 1975)
- PP 139  
Morgan, William F., Jr., "Beyond Folklore and Fables in Forestry to Positive Economics," 14 pp., (Presented at Southern Economic Association Meetings November, 1974) Aug 1975, AD A 015 293
- PP 140  
Mahoney, Robert and Druckman, Daniel, "Simulation, Experimentation, and Context," 36 pp., 1 Sep 1975, (Published in Simulation & Games, Vol. 6, No. 3, Sep 1975)  
\*Mathematica, Inc.
- PP 141  
Mizrahi, Maurice M., "Generalized Hermite Polynomials," 5 pp., Feb 1976 (Reprinted from the Journal of Computational and Applied Mathematics, Vol. 1, No. 4 (1975), 273-277).  
\*Research supported by the National Science Foundation
- PP 142  
Lockman, Robert F., Jehn, Christopher, and Shughart, William F. II, "Models for Estimating Premature Losses and Recruiting District Performance," 36 pp., Dec 1975 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A 020 443
- PP 143  
Horowitz, Stanley and Sherman, Allan (LCdr., USN), "Maintenance Personnel Effectiveness in the Navy," 33 pp., Jan 1976 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A021 581
- PP 144  
Durch, William J., "The Navy of the Republic of China - History, Problems, and Prospects," 66 pp., Aug 1976 (To be published in "A Guide to Asiatic Fleets," ed. by Barry M. Blechman; Naval Institute Press) AD A030 460
- PP 145  
Kelly, Anne M., "Port Visits and the "Internationalist Mission" of the Soviet Navy," 36 pp., Apr 1976 AD A023 436
- PP 146  
Palmour, Vernon E., "Alternatives for Increasing Access to Scientific Journals," 6 pp., Apr 1975 (Presented at the 1975 IEEE Conference on Scientific Journals, Cherry Hill, N.C., Apr 28-30; published in IEEE Transactions on Professional Communication, Vol. PC-18, No. 3, Sep 1975) AD A021 798
- PP 147  
Kessler, J. Christian, "Legal Issues in Protecting Offshore Structures," 33 pp., Jun 1976 (Prepared under task order N00014-68-A-0091-0023 for ONR) AD A028 389
- PP 148  
McConnell, James M., "Military-Political Tasks of the Soviet Navy in War and Peace," 62 pp., Dec 1975 (Published in Soviet Oceans Development Study of Senate Commerce Committee October 1976) AD A022 580
- PP 149  
Squires, Michael L., "Counterforce Effectiveness: A Comparison of the Tsipis "K" Measure and a Computer Simulation," 24 pp., Mar 1976 (Presented at the International Study Association Meetings, 27 Feb 1976) AD A022 591
- PP 150  
Kelly, Anne M. and Petersen, Charles, "Recent Changes in Soviet Naval Policy: Prospects for Arms Limitations in the Mediterranean and Indian Ocean," 28 pp., Apr 1976, AD A 023 723
- PP 151  
Horowitz, Stanley A., "The Economic Consequences of Political Philosophy," 8 pp., Apr 1976 (Reprinted from Economic Inquiry, Vol. XIV, No. 1, Mar 1976)
- PP 152  
Mizrahi, Maurice M., "On Path Integral Solutions of the Schrodinger Equation, Without Limiting Procedure," 10 pp., Apr 1976 (Reprinted from Journal of Mathematical Physics, Vol. 17, No. 4 (Apr 1976), 566-575).  
\*Research supported by the National Science Foundation
- PP 153  
Mizrahi, Maurice M., "WKB Expansions by Path Integrals, With Applications to the Anharmonic Oscillator," 137 pp., May 1976, AD A025 440  
\*Research supported by the National Science Foundation
- PP 154  
Mizrahi, Maurice M., "On the Semi-Classical Expansion in Quantum Mechanics for Arbitrary Hamiltonians," 19 pp., May 1976 (Published in Journal of Mathematical Physics, Vol. 18, No. 4, p. 786, Apr 1977), AD A025 441
- PP 155  
Squires, Michael L., "Soviet Foreign Policy and Third World Nations," 26 pp., Jun 1976 (Prepared for presentation at the Midwest Political Science Association meetings, Apr 30, 1976) AD A028 388
- PP 156  
Stallings, William, "Approaches to Chinese Character Recognition," 12 pp., Jun 1976 (Reprinted from Pattern Recognition (Pergamon Press), Vol. B, pp. 87-98, 1976) AD A028 692
- PP 157  
Morgan, William F., "Unemployment and the Pentagon Budget: Is There Anything in the Empty Pork Barrel?" 20 pp., Aug 1976 AD A030 455
- PP 158  
Haskell, LCdr. Richard D. (USN), "Experimental Validation of Probability Predictions," 25 pp., Aug 1976 (Presented at the Military Operations Research Society Meeting, Fall 1976) AD A030 458
- PP 159  
McConnell, James M., "The Gorskov Articles, The New Gorskov Book and Their Relation to Policy," 93 pp., Jul 1976 (Published in Soviet Naval Influence: Domestic and Foreign Dimensions, ed. by M. McGwire and J. McDonnell; New York: Praeger, 1977) AD A029 227
- PP 160  
Wilson, Desmond P., Jr., "The U.S. Sixth Fleet and the Conventional Defense of Europe," 60 pp., Sep 1976 (Submitted for publication in Adelphi Papers, I.I.S.S., London) AD A030 457
- PP 161  
Melich, Michael E. and Peet, Vice Adm. Ray (USN, Retired), "Fleet Commanders: Afloat or Ashore?" 9 pp., Aug 1976 (Reprinted from U.S. Naval Institute Proceedings, Jun 1976) AD A030 456
- PP 162  
Friedheim, Robert L., "Parliamentary Diplomacy," 106 pp., Sep 1976 AD A033 305
- PP 163  
Lockman, Robert F., "A Model for Predicting Recruit Losses," 9 pp., Sep 1976 (Presented at the 84th annual convention of the American Psychological Association, Washington, D.C., 4 Sep 1976) AD A030 459
- PP 164  
Mahoney, Robert B., Jr., "An Assessment of Public and Elite Perceptions in France, The United Kingdom, and the Federal Republic of Germany, 31 pp., Feb 1977 (Presented at Conference "Perception of the U.S. - Soviet Balance and the Political Uses of Military Power" sponsored by Director, Advanced Research Projects Agency, April 1976) AD 036 599
- PP 165  
Jondrow, James M., "Effects of Trade Restrictions on Imports of Steel," 67 pp., November 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 166  
Feldman, Paul, "Impediments to the Implementation of Desirable Changes in the Regulation of Urban Public Transportation," 12 pp., Oct 1976, AD A033 322
- PP 166 - Revised  
Feldman, Paul, "Why It's Difficult to Change Regulation," Oct 1976
- PP 167  
Kleinman, Samuel, "ROTC Service Commitments: a Comment," 4 pp., Nov 1976, (To be published in Public Choice, Vol. XXIV, Fall 1976) AD A033 305
- PP 168  
Lockman, Robert F., "Revalidation of CNA Support Personnel Selection Measures," 36 pp., Nov 1976
- PP 169  
Jacobson, Louis S., "Earnings Losses of Workers Displaced from Manufacturing Industries," 38 pp., Nov 1976, (Delivered at ILAB Conference in Dec 1976), AD A039 809
- PP 170  
Brechling, Frank P., "A Time Series Analysis of Labor Turnover," Nov 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 171  
Ralston, James M., "A Diffusion Model for GaP Red LED Degradation," 10 pp., Nov 1976, (Published in Journal of Applied Physics, Vol. 47, pp. 4518-4527, Oct 1976)



- PP 172  
Classen, Kathleen P., "Unemployment Insurance and the Length of Unemployment," Dec 1976. (Presented at the University of Rochester Labor Workshop on 16 Nov 1976)
- PP 173  
Kleinman, Samuel D., "A Note on Racial Differences in the Added-Worker/Discouraged-Worker Controversy," 2 pp., Dec 1976. (Published in the *American Economist*, Vol. XX, No. 1, Spring 1976)
- PP 174  
Mahoney, Robert B., Jr., "A Comparison of the Brookings and International Incidents Projects," 12 pp. Feb 1977 AD 037 206
- PP 175  
Levine, Daniel; Stoloff, Peter and Spruill, Nancy, "Public Drug Treatment and Addict Crime," June 1976. (Published in *Journal of Legal Studies*, Vol. 5, No. 2)
- PP 176  
Felix, Wendi, "Correlates of Retention and Promotion for USNA Graduates," 38 pp., Mar 1977, AD A039 040
- PP 177  
Lockman, Robert F. and Warner, John T., "Predicting Attrition: A Test of Alternative Approaches," 33 pp. Mar 1977. (Presented at the OSD/ONR Conference on Enlisted Attrition Xerox International Training Center, Leesburg, Virginia, 4-7 April 1977), AD A039 047
- PP 178  
Kleinman, Samuel D., "An Evaluation of Navy Unrestricted Line Officer Accession Programs," 23 pp. April 1977. (To be presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 048
- PP 179  
Stoloff, Peter H. and Balut, Stephen J., "Vacate: A Model for Personnel Inventory Planning Under Changing Management Policy," 14 pp. April 1977. (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 049
- PP 180  
Horowitz, Stanley A. and Sherman, Allen, "The Characteristics of Naval Personnel and Personnel Performance," 16 pp. April 1977. (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 060
- PP 181  
Balut, Stephen J. and Stoloff, Peter, "An Inventory Planning Model for Navy Enlisted Personnel," 35 pp., May 1977. (Prepared for presentation at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Science, 9 May 1977, San Francisco, California), AD A042 221
- PP 182  
Murray, Russell, 2nd, "The Quest for the Perfect Study or My First 1136 Days at CNA," 57 pp., April 1977
- PP 183  
Kassing, David, "Changes in Soviet Naval Forces," 33 pp., November, 1976. (Published as part of Chapter 3, "General Purpose Forces: Navy and Marine Corps," in *Arms, Men, and Military Budgets*, Francis P. Hoebler and William Schneider, Jr. (eds.), (Crane, Russak & Company, Inc.: New York), 1977), AD A040 106
- PP 184  
Lockman, Robert F., "An Overview of the OSD/ONR Conference on First Term Enlisted Attrition," 22 pp., June 1977. (Presented to the 38th MORS Working Group on Manpower and Personnel Planning, Annapolis, Md., 28-30 June 1977), AD A043 618
- PP 185  
Kassing, David, "New Technology and Naval Forces in the South Atlantic," 22 pp. (This paper was the basis for a presentation made at the Institute for Foreign Policy Analyses, Cambridge, Mass., 28 April 1977), AD A043 619
- PP 186  
Mizrahi, Maurice M., "Phase Space Integrals, Without Limiting Procedure," 31 pp., May 1977. (Invited paper presented at the 1977 NATO Institute on Path Integrals and Their Application in Quantum Statistical, and Solid State Physics, Antwerp, Belgium, July 17-30, 1977) (Published in *Journal of Mathematical Physics* 19(1), p. 296, Jan 1978), AD A040 107
- PP 187  
Coile, Russell C., "Nomography for Operations Research," 35 pp., April 1977. (Presented at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Science, San Francisco, California, 9 May 1977), AD A043 620
- PP 188  
Durch, William J., "Information Processing and Outcome Forecasting for Multilateral Negotiations: Testing One Approach," 53 pp., May 1977. (Prepared for presentation to the 18th Annual Convention of the International Studies Association, Chase Park Plaza Hotel, St. Louis, Missouri, March 18-20, 1977), AD A042 222
- PP 189  
Coile, Russell C., "Error Detection in Computerized Information Retrieval Data Bases," July, 1977, 13 pp. Presented at the Sixth Cranfield International Conference on Mechanized Information Storage and Retrieval Systems, Cranfield Institute of Technology, Cranfield, Bedford, England, 26-29 July 1977, AD A043 580
- PP 190  
Mahoney, Robert B., Jr., "European Perceptions and East-West Competition," 96 pp., July 1977. (Prepared for presentation at the annual meeting of the International Studies Association, St. Louis, Mo., March, 1977), AD A043 661
- PP 191  
Sawyer, Ronald, "The Independent Field Assignment: One Man's View," August 1977, 25 pp.
- PP 192  
Holen, Arlene, "Effects of Unemployment Insurance Entitlement on Duration and Job Search Outcome," August 1977, 6 pp., (Reprinted from *Industrial and Labor Relations Review*, Vol. 30, No. 4, Jul 1977)
- PP 193  
Horowitz, Stanley A., "A Model of Unemployment Insurance and the Work Test," August 1977, 7 pp. (Reprinted from *Industrial and Labor Relations Review*, Vol. 30, No. 40, Jul 1977)
- PP 194  
Classen, Kathleen P., "The Effects of Unemployment Insurance on the Duration of Unemployment and Subsequent Earnings," August 1977, 7 pp. (Reprinted from *Industrial and Labor Relations Review*, Vol. 30, No. 40, Jul 1977)
- PP 195  
Brechtling, Frank, "Unemployment Insurance Taxes and Labor Turnover: Summary of Theoretical Findings," 12 pp. (Reprinted from *Industrial and Labor Relations Review*, Vol. 30, No. 40, Jul 1977)
- PP 196  
Reiston, J. M. and Lorimer, O. G., "Degradation of Bulk Electroluminescent Efficiency in Zn, O-Doped GaP LED's," July 1977, 3 pp. (Reprinted from *IEEE Transactions on Electron Devices*, Vol. ED-24, No. 7, July 1977)
- PP 197  
Wells, Anthony R., "The Centre for Naval Analyses," 14 pp., Dec 1977, AD A049 107
- PP 198  
Classen, Kathleen P., "The Distributional Effects of Unemployment Insurance," 25 pp., Sept. 1977. (Presented at a Hoover Institution Conference on Income Distribution, Oct 7-8, 1977)
- PP 199  
Durch, William J., "Revolution From A.F.A.R. - The Cuban Armed Forces in Africa and the Middle East," Sep 1977, 16 pp., AD A046 268
- PP 200  
Powers, Bruce F., "The United States Navy," 40 pp. Dec 1977. (To be published as a chapter in *The U.S. War Machine by Salamander Books in England during 1978*), AD A049 108
- PP 201  
Durch, William J., "The Cuban Military in Africa and The Middle East: From Algeria to Angola," Sep 1977, 67 pp., AD A045 676
- PP 202  
Feldman, Paul, "Why Regulation Doesn't Work," (Reprinted from *Technological Change and Welfare in the Regulated Industries and Review of Social Economy*, Vol. XXIX, March, 1971, No. 1.) Sep 1977, 8 pp.
- PP 203  
Feldman, Paul, "Efficiency, Distribution, and the Role of Government in a Market Economy," (Reprinted from *The Journal of Political Economy*, Vol. 79, No. 3, May/June 1971.) Sep 1977, 19 pp., AD A045 675

PP 204

Wells, Anthony R., "The 1967 June War: Soviet Naval Diplomacy and The Sixth Fleet - A Reappraisal," Oct 1977, 36 pp., AD A047 236

PP 205

Coile, Russell C., "A Bibliometric Examination of the Square Root Theory of Scientific Publication Productivity," (Presented at the annual meeting of the American Society for Information Science, Chicago, Illinois, 29 September 1977.) Oct 1977, 6 pp., AD A047 237

PP 206

McConnell, James M., "Strategy and Missions of the Soviet Navy in the Year 2000," 48 pp., Nov 1977, (Presented at a Conference on Problems of Sea Power as we Approach the 21st Century, sponsored by the American Enterprise Institute for Public Policy Research, 6 October 1977, and subsequently published in a collection of papers by the Institute), AD A047 244

PP 207

Goldberg, Lawrence, "Cost-Effectiveness of Potential Federal Policies Affecting Research & Development Expenditures in the Auto, Steel and Food Industries," 36 pp., Oct 1977, (Presented at Southern Economic Association Meetings beginning 2 November 1977)

PP 208

Roberts, Stephen S., "The Decline of the Overseas Station Fleets: The United States Asiatic Fleet and the Shanghai Crisis, 1932," 18 pp., Nov 1977, (Reprinted from *The American Neptune*, Vol. XXXVII, No. 3, July 1977), AD A047 245

PP 209 - Classified

PP 210

Kassing, David, "Protecting The Fleet," 40 pp., Dec 1977 (Prepared for the American Enterprise Institute Conference on Problems of Sea Power as We Approach the 21st Century, October 6-7, 1977), AD A048 108

PP 211

Mizrahi, Maurice M., "On Approximating the Circular Coverage Function," 14 pp., Feb 1978

PP 212

Mangel, Marc, "On Singular Characteristic Initial Value Problems with Unique Solutions," 20 pp., Jun 1978 (To be submitted for publication in *Journal of Mathematical Analysis and Its Applications*)

PP 213

Mangel, Marc, "Fluctuations in Systems with Multiple Steady States. Application to Lancaster Equations," 12 pp., Feb 78, (Presented at the First Annual Workshop on the Information Linkage Between Applied Mathematics and Industry, Naval PG School, Feb 23-25, 1978)

PP 214

Wainland, Robert G., "A Somewhat Different View of The Optimal Naval Posture," 37 pp., Jun 1978 (Presented at the 1978 Convention of the American Political Science Association (APSA/IUS Panel on "Changing Strategic Requirements and Military Posture"), Chicago, Ill., September 2, 1978)

PP 215

Coile, Russell C., "Comments on: *Principles of Information Retrieval* by Manfred Kochen, 10 pp., Mar 78, (Published as a Letter to the Editor, *Journal of Documentation*, Vol. 31, No. 4, pages 298-301, December 1975)

PP 216

Coile, Russell C., "Lotka's Frequency Distribution of Scientific Productivity," 18 pp., Feb 1978, (Published in the *Journal of the American Society for Information Science*, Vol. 28, No. 6, pp. 366-370, November 1977)

PP 217

Coile, Russell C., "Bibliometric Studies of Scientific Productivity," 17 pp., Mar 78, (Presented at the Annual meeting of the American Society for Information Science held in San Francisco, California, October 1976.)

PP 218 - Classified

PP 219

Huntzinger, R. LaVar, "Market Analysis with Rational Expectations: Theory and Estimation," 60 pp., Apr 78 (To be submitted for publication in *Journal of Econometrics*)

PP 220

Maurer, Donald E., "Diagonalization by Group Matrices," 26 pp., Apr 78

PP 221

Weinland, Robert G., "Superpower Naval Diplomacy in the October 1973 Arab-Israeli War," 76 pp., Jun 1978

PP 222

Mizrahi, Maurice M., "Correspondence Rules and Path Integrals," 30 pp., Jun 1978 (Invited paper presented at the CNRS meeting on "Mathematical Problems in Feynman's Path Integrals," Marseille, France, May 22-26, 1978)

PP 223

Mangel, Marc, "Stochastic Mechanics of Molecule-Molecule Reactions," 21 pp., Jun 1978 (To be submitted for publication in *Journal of Mathematical Physics*)

PP 224

Mangel, Marc, "Aggregation, Bifurcation, and Extinction in Exploited Animal Populations," 48 pp., Mar 1978 (To be submitted for publication in *American Naturalist*)  
"Portions of this work were started at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, B.C., Canada

PP 225

Mangel, Marc, "Oscillations, Fluctuations, and the Hopf Bifurcation," 43 pp., Jun 1978  
"Portions of this work were completed at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, Canada

PP 226

Ralston, J. M. and J. W. Mann, "Temperature and Current Dependence of Degradation in Red-Emitting GaP LEDs," 34 pp., Jun 1978

PP 227

Mangel, Marc, "Uniform Treatment of Fluctuations at Critical Points," 50 pp., May 1978 (To be submitted for publication in *Journal of Statistical Physics*)

PP 228

Mangel, Marc, "Relaxation at Critical Points: Deterministic and Stochastic Theory," 54 pp., Jun 1978 (To be submitted for publication in *Journal of Mathematical Physics*)

PP 229

Mangel, Marc, "Diffusion Theory of Reaction Rates. I: Formulation and Einstein-Smoluchowski Approximation," 50 pp., Jan 1978

PP 230

Mangel, Marc, "Diffusion Theory of Reaction Rates. II: Ornstein-Uhlenbeck Approximation," 34 pp., Feb 1978

PP 231

Wilson, Desmond P., Jr., "Naval Projection Forces: The Case for a Responsive MAF," Aug 1978

PP 232

Jacobson, Louis, "Can Policy Changes be Made Acceptable to Labor?" Aug 1978 (To be submitted for publication in *Industrial and Labor Relations Review*)

PP 233

Jacobson, Louis, "An Alternative Explanation of the Cyclical Pattern of Quits," 23 pp., Sep 1978

PP 234

Jondrow, James and Levy, Robert A., "Does Federal Expenditure Displace State and Local Expenditure: The Case of Construction Grants," 18 pp., Oct 1978 (To be submitted for publication in *Journal of Public Economics*)

PP 235

Mizrahi, Maurice M., "The Semiclassical Expansion of the Anharmonic-Oscillator Propagator," 41 pp., Oct 1978 (To appear in the *Journal of Mathematical Physics*)

PP 237

Maurer, Donald, "A Matrix Criterion for Normal Integral Bases," 10 pp., Jan 1979

PP 238

Utgoft, Kathleen Classen, "Unemployment Insurance and The Employment Rate," 20 pp., Oct 1978

PP 239

Trost, R. P. and Warner, J. T., "The Effects of Military Occupational Training on Civilian Earnings: An Income Selectivity Approach," 38 pp., Nov 1979 (To be submitted for publication in *Review of Economics and Statistics*)

PP 240

Powers, Bruce, "Goals of the Center for Naval Analyses," 13 pp., Dec 1978

PP 241

Mangel, Marc, "Fluctuations at Chemical Instabilities," 24 pp., Dec 1978 (Published in *Journal of Chemical Physics*, Vol. 69, No. 8, Oct 15, 1978)

- PP 242  
Simpson, William R., "The Analysis of Dynamically Interactive Systems (Air Combat by the Numbers)," 160 pp., Dec 1978
- PP 243  
Simpson, William R., "A Probabilistic Formulation of Murphy Dynamics as Applied to the Analysis of Operational Research Problems," 18 pp., Dec 1978 (Submitted for publication in *The Journal of Irreproducible Results*)
- PP 244  
Sherman, Allan and Horowitz, Stanley A., "Maintenance Costs of Complex Equipment," 20 pp., Dec 1978
- PP 245  
Simpson, William R., "The Accelerometer Methods of Obtaining Aircraft Performance from Flight Test Data (Dynamic Performance Testing)," 403 pp., Jun 1979
- PP 248  
Thomas, James A., Jr., "The Transport Properties of Dilute Gases in Applied Fields," 183 pp., Mar 1979
- PP 249  
Glasser, Kenneth S., "A Secretary Problem with a Random Number of Choices," 23 pp., Mar 1979 (Submitted for publication in *Journal of the American Statistical Association*)
- PP 250  
Mangel, Marc, "Modeling Fluctuations in Macroscopic Systems," 26 pp., Jun 1979
- PP 251  
Trost, Robert P., "The Estimation and Interpretation of Several Selectivity Models," 37 pp., Jun 1979
- PP 252  
Nunn, Walter R., "Position Finding with Prior Knowledge of Covariance Parameters," 5 pp., Jun 1979
- PP 253  
Glasser, Kenneth S., "The d Choice Secretary Problem," 32 pp., Jun 1979
- PP 254  
Mangel, Marc and Quanbeck, David B., "Integration of a Bivariate Normal Over an Offset Circle," 14 pp., Jun 1979
- PP 255 - Classified
- PP 257  
Thaler, R., "Discounting and Fiscal Constraints: Why Discounting is Always Right," 10 pp., Aug 1979
- PP 258  
Mangel, Marc S. and Thomas, James A., Jr., "Analytical Methods in Search Theory," 86 pp., Nov 1979
- PP 259  
Glass, David V.; Hsu, Ih-Ching; Nunn, Walter R. and Perin, David A., "A Class of Commutative Markov Matrices," 17 pp., Nov 1979 (To be submitted for publication in *Operations Research*)
- PP 260  
Mangel, Marc S. and Cope, David K., "Detection Rate and Sweep Width in Visual Search," 14 pp., Nov 1979
- PP 261  
Vile, Carlos L.; Zvijec, David J. and Ross, John, "Franck-Condon Theory of Chemical Dynamics. VI. Angular Distributions of Reaction Products," 14 pp., Nov 1979 (Reprinted from *Journal Chem. Phys.* 70(12), 15 Jun 1979)
- PP 262  
Petersen, Charles C., "Third World Military Elites in Soviet Perspective," 50 pp., Nov 1979 (To be submitted for publication in *International Security*)